

Comparative Assessment of Three Models for Estimating Weibull Parameters for Wind Energy Applications in a Nigerian Location

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Abstract-The two-parameter Weibull distribution has been commonly used, accepted and recommended in literature to express the wind speed frequency distribution for most wind locations. Consequently, the shape and scale parameters of the distribution are frequently used to design and characterize commercial wind conversion machines. In this paper, we examine three different models commonly used for estimating these parameters in order to determine the model that is best suited for Nigerian locations using 3-year wind data for Nsukka (6.8°N; 7.4°E). The coefficient of determination and root-mean-square error of the three models are compared for the location. Results show that the moment model appears more appropriate for estimating these parameters for the location.

Keywords-renewable energy; wind - frequency distribution

I INTRODUCTION

Wind is an effect caused by motion of air in the atmosphere driven by pressure differences over heights and regions. It is estimated that between 1.5 to 2.5% of the global solar radiation received on the surface of the earth is converted to wind (Vosburgh, 1983). The force carried by the wind can be harnessed for useful purposes such as grinding grain and generating electricity (in windmills). Hence, wind energy, which contributes very little pollution and few greenhouse gases to the environment, is a valuable alternative to non-renewable fossil fuels (Taylor, 1983). Wind energy application is recently described as an economic attractive solution to the urgent electrification problem of many countries and is the world's fastest growing energy source, expanding at annual rate of 25 to 35% (Weisser and Garcia 2006). However, detailed knowledge of wind characteristics and speed distribution over time in a given location is crucial in selecting/designing optimum wind energy conversion systems to optimize energy output and minimize utility generation cost (Pallabazzer, 2003).

In previous papers (Odo et al. 2009; 2010), theoretical potentials of wind in Nsukka and Enugu were examined based on annual mean wind speed. However, for reliable assessment of wind potentials and characteristics of any location, long term observation of wind speed frequency distribution is necessary. Wind speed is a real valued random variable and its distribution over time is represented by probability density

functions of various forms and the extent to which wind can be exploited as a source of energy depends on the probability density of occurrence of different speeds at the site (e.g. Enibe 1987; Walker and Jenkins 1997). Among the probability density functions that have been proposed for wind speed frequency distribution of most locations, the Weibull function has been the most acceptable distribution and forms the basis for commercial wind energy applications and software (Seyit and Ali 2009), such as the Wind Atlas Analysis and Application Program (WAsP) and the recently developed Nigerian Wind Energy Information System (WIS) software.

The Weibull probability density function is a two-parameter function characterized by a dimensionless shape (k) and scale (c) parameters. These two parameters determine the wind speed for optimum performance of a wind conversion system as well as the speed range over which the device is likely to operate (Ramachandra et al. 2005). It is therefore, very essential to accurately estimate the parameters for any candidate site for installation of wind energy conversion systems. Various models have been proposed for estimating the parameters and the suitability of each model varies with sample data distribution, which essentially, is location specific. In this paper, three frequently used models are examined and their suitability compared for Nsukka location.

II MATERIALS AND METHOD

In this study, we use 3-year (2008 – 2010) wind data for Nsukka (6.8°N; 7.4°E) at 10m meteorological height, obtained from the Centre for Basic Space Science (CBSS), University of Nigeria, Nsukka. The data give information on wind speed distribution, in five (5) minutes intervals, over the study period. However, we find that the data recovery rate for the location is 94.1%.

A. The Weibull Probability Density Function.

Wind power developers measure actual wind resources, in part, to determine the distribution of wind speed for the location, because of its considerable influence on wind potential. The Weibull probability density function is a mathematical idealization of the distribution of wind speed over time. The function gives the probability of the wind speed

being in 1 m/s interval centred on a particular speed (v), taking into account both seasonal and annual variations over the period covered by the statistics. The Weibull distribution function is given (e.g. Walker and Jenkins, 1997; Gipe, 2004) by:

$$f_{(v)} = \frac{k}{c} \left(\frac{v}{c} \right)^{k-1} \exp \left(- \left(\frac{v}{c} \right)^k \right) \quad (1)$$

where $f(v)$ is the probability density defined as the frequency of occurrence of wind speed (v), c (in unit of m/s) is the scale parameter which is closely related to the modal wind speed for the location, and k is the dimensionless shape parameter which describes the form and width of the distribution. The Weibull distribution is therefore characterized by the two parameters c and k . The cumulative Weibull distribution $F(v)$ which gives the probability of the wind speed exceeding the value v is expressed (Justus et al. 1978; Walker and Jenkins, 1997) as:

$$F_{(v)} = \exp \left(- \left(\frac{v}{c} \right)^k \right) \quad (2)$$

On the other hand, the power derivable from the wind is a cubic function of the wind speed, such that, in the Weibull distribution, the power density (P_A) of the wind at any speed is given (e.g. Walker and Jenkins, 1997) by:

$$P_{(A)} = \frac{1}{2} \rho \int_0^\infty v^3 f_{(v)} dv \quad (3)$$

where ρ is the density of air. Analysis of equation (3) using equation (1) shows that the power density could be expressed as a gamma function (Γ) defined in x -variable (e.g. Dass, 1998) as:

$$\Gamma x = \int_0^\infty x^{n-1} e^{-x} dx \quad (4)$$

Using equation (4) for v in equation (3), the average wind power density (P_{av}) based on Weibull distribution can be expressed (e.g. Ucar and Baló, 2009) in the form:

$$P_{(av)} = \frac{1}{2} \rho \overline{v^3} \frac{\Gamma \left(1 + \frac{3}{k} \right)}{\left[\Gamma \left(1 + \frac{1}{k} \right) \right]^3} \quad (5)$$

where $\overline{v^3}$ is the mean of the cube of wind speed distribution. However, meteorologists have characterized the wind speed distribution patterns for many of the world's wind regimes. In temperate climate (mid latitudes), a typical shape parameter (k) of 2 offers a good approximation (Enibe, 1987; Gipe, 2004). For $k = 2$, equation (1) or (2) reduces to Raleigh wind speed distribution. Hence, the Raleigh distribution is a special case of the Weibull distribution developed for estimation of wind potential in temperate climate locations. Wind characteristics are essentially location specific and performance of practical wind conversion devices may greatly differ if actual wind conditions at the location differ from those standard speed distributions. We examine three models commonly used for estimating these Weibull parameters for Nsukka location in the next section.

B. Models for estimating Weibull parameters

Estimation of the Weibull distribution parameters correctly for any location is essential in optimizing the design of wind energy conversion devices to maximize energy extraction.

Several models have been proposed to estimate these wind speed parameters for different locations. The frequently used models include the Graphic (Jaramillo and Borja 2004), the Moment (e.g. Justus et al. 1978; Gipe 2004) and the Power density/Energy pattern (Seyit and Ali 2009) models. However, it has been noted (Seyit and Ali 2009) that the suitability of each of the model varies with sample data distribution, which, nevertheless, varies from location to location. In this section, these three models are outlined.

1) Graphic Model

This model is an analytical method derived using the cumulative Weibull distribution function given in equation (2). It is the most fundamental model derived by taking the logarithm of equation (2) twice to yield:

$$\ln \{-\ln F_{(v)}\} = \ln v - \ln c \quad (6)$$

Equation (6) is a linear equation in $F(v) - v$ plane and one-dimensional regression analysis of $\ln \{-\ln F(v)\}$ against $\ln v$ plot for any location gives k and c from the slope and intercept respectively. This method involves sorting the wind speed data into bins, calculating cumulative frequency probability for each bin, and finally, solving the linear least square problem, to find the wind speed distribution parameters.

2) Moment model

This model was first proposed by Justus et al. (1978) using the mean wind speed (μ) and standard deviation (σ) for a given distribution. In the context of this model, the shape and scale parameters of the wind speed distribution are respectively given (e.g. Seyit and Ali 2009; Iheonu et al. 2002) by:

$$k = \left(\frac{\sigma}{\mu} \right)^{-1.086} \quad (7)$$

and

$$c = \frac{\mu}{\Gamma \left(1 + \frac{1}{k} \right)} \quad (8)$$

This model also requires the long process of summation of the wind speed data to find the mean value and standard deviation, and subsequently, estimating k and c numerically, in terms of equations (7) and (8) respectively.

3) Energy pattern/Power density model.

This is the most recent model developed by Seyit and Ali (2009) for estimating the wind speed distribution shape (k) factor. According to this model, k is given by:

$$k = 1 + \frac{3.69}{E_f^2} \quad (9)$$

where E_f is the energy pattern factor defined (e.g. Gipe, 2004) as the ratio of the power density derived from speed distribution to that derived from average speed and given (Gipe, 2004; Seyit and Ali 2009) by:

$$E_f = \frac{\overline{v^3}}{\mu^3} = \frac{\Gamma \left(1 + \frac{3}{k} \right)}{\left[\Gamma \left(1 + \frac{1}{k} \right) \right]^3} \quad (10)$$

Similarly, the scale (c) parameter can be obtained by numerical solution of equation (8) for the value of k .

III RESULTS AND DISCUSSION

We obtain the annual mean wind speed (μ) at 10m height for the location as 1.35m/s with standard deviation (σ) of 0.79m/s. On the other hand, the mean of the cube of the distribution ($\overline{v^3}$) is 5.33m³/s³. For each of the three models described in section 2.2, we derive the Weibull k and c parameters as appropriate. The results are as shown in table 1.

Furthermore, we use the results in each case to predict the probability densities for different wind speed bins and the results are compared with measured values. The measured and predicted values are displayed in table 2, while the plots of the probability density distributions for measured and estimated values are shown in Figure 1. The suitability of each of the models in predicting wind speed probability density for the location was determined using two independent statistics, namely, the coefficient of determination (R^2) and root-mean-square error (RMSE).

The coefficient of determination is a non parametric statistic which determines the degree of association between two random variables and is defined (e.g. Aalen, 1978) as:

$$R^2 = 1 - \frac{\sum_{i=1}^N (y_i - x_i)^2}{\sum_{i=1}^N (y_i - \bar{y})^2} \quad (11)$$

This statistic varies from 0 (for a null association) to 1 (for a perfect association). On the other hand, RMSE is a statistical quantity which determines the degree of departure of two data sets from a supposed association and is defined numerically (e.g. Aalen 1978: Joanes and Gill 1998) by:

$$RMSE = \left[\frac{1}{N} \sum (y_i - x_i)^2 \right]^{\frac{1}{2}} \quad (12)$$

where N is the number of data points in each set, y_i and x_i are

the corresponding values in the two data sets, while \bar{y} is the mean of y data. Thus, the model with larger value of R^2 and/or smaller value of RMSE should, in principle, give more accurate estimate of the wind potential for the location. Both R^2 and RMSE were determined for each model and the results are as displayed in table 1.

TableI COMPARISON OF THE THREE MODELS OF ESTIMATING WEIBULL PARAMETERS FOR NSUKKA

Parameters	Graphic model	Moment model	Energy pattern model
K	2.56	2.53	1.79
c (ms ⁻¹)	2.90	1.90	1.36
R^2	0.283	0.993	0.566
RMSE	0.088	0.014	0.085

A cursory look at Figure (1) shows obviously that a major difference between the distributions lies on the value and position (on v – axis) of the peak of the distribution, with

Moment distribution matching the measured values more closely. These are two important factors in designing a wind turbine for a particular location. While the position on v-axis (corresponding to c-parameter) gives the characteristic wind speed for which a turbine must be designed for maximum output; the peak value gives information on the total time the turbine is expected to operate. Hence, for a maximum output of a wind turbine, its design should be matched with the wind speed that corresponds to the peak of the probability density distribution (Walker and Jenkins, 1997). The characteristic wind speeds at the peak of the distributions are 1.9, 2.9, 1.9 and 1.4m/s respectively for Measured, Graphic, Moment and Energy pattern distributions at 10m meteorological height. Another difference observed in the distributions is the width of the distributions. This however, depends on the k-parameter and gives information on the range of speed for which a wind conversion machine designed for the mid-speed (c) will operate. Figure (1) shows that Graphic distribution has largest width, while Energy pattern has the smallest width. Furthermore, the width of the Moment distribution closely matches that of measured data. On the other hand, results of the non-parametric analyses for these models show that Moment model has the highest R^2 (~ 0.993) and least RMSE (~ 0.014).

The Graphic model has the least R^2 (~ 0.283) and largest RMSE (~ 0.088). These observations suggest that the moment model may be more reliable in estimating wind potential for Nsukka location.

TableII COMPARISON OF PROBABILITY DENSITY DISTRIBUTIONS OF THE THREE MODELS

Wind speed (m/s)	Measured value (%)	Graphic model (%)	Moment model (%)	Energy pattern model (%)
0.5	19.2	5.6	16.7	51.1
1.0	39.9	15.7	41.0	58.3
1.5	48.5	26.2	53.5	43.1
2.0	42.2	33.6	46.1	24.0
2.5	27.6	35.3	27.4	10.6
3.0	13.9	31.3	11.2	3.8
3.5	5.4	23.5	3.1	1.2
4.0	1.6	14.9	0.6	0.3
4.5	0.4	8.1	0.1	0.1
5.0	0.0	3.6	0.0	0.0
5.5	0.0	1.4	0.0	0.0
6.0	0.0	0.4	0.0	0.0
6.5	0.0	0.1	0.0	0.0
7.0	0.0	0.0	0.0	0.0

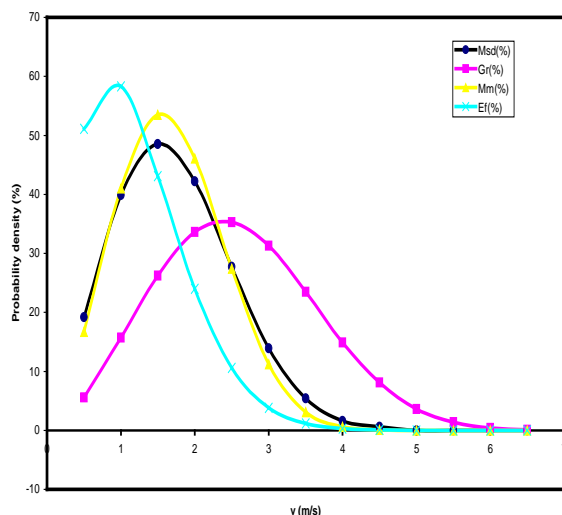


Fig 1. Comparison of probability density distributions of the methods.
Msd = measured; Gr =Graphic; Mm = Moment; Ef = Energy pattern.

IV CONCLUSION

We have assessed three different models used in estimating Weibull distribution parameters for wind energy applications in Nsukka. The models are Graphic, Moment and Energy pattern models. Results show that Moment model gives more accurate prediction of wind potentials for the location as it gives smaller value of root-mean-square error and higher coefficient of determination than the other two models. The results therefore, strongly suggest that Moment model may be more reliable in estimating these parameters for the Nsukka location. However, extension of this study to other locations in Nigeria and over longer study periods is required for a sound conclusion.

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